Just the Facts:
An Informed Response
to the Invalid Research Findings
in “From Learning Multiplication
Facts to Automaticity”

(W 08 issue of ***The Educational Therapist*** by Peg W. Bogle, MBA, MEd)

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David Berg prefaces his rebuttal to Peg W. Bogle’s article with the following:

> *In the winter 2008 issue of* The Educational Therapist *Peg W. Bogle compares my method, the* 9 Lines multiplication fact acquisition and application strategy *(referred to in the following article as:* 9 Lines Strategy*), with the use of flash cards. Bogle’s misapplication of the* 9 Lines strategy, *in conjunction with the flawed design of her study, render her conclusions invalid, as I will explain in my rebuttal article.*

In my experience, the stamp of “research-based” provides the principal criterion for determining whether a program, a method, or an idea is valid and/or proven. The banner of “research-based” is waved with increasing vigor these days, and in theory, the scientific validation of a research basis should provide the standards on which we can depend. For research to be valid, it must be performed with scientific integrity, and in the case of a study comparing methodologies, the methods used in the study must be applied correctly to support the conclusions and findings therein. I believe the term *research-based* is becoming so unduly seductive, that rather than subjecting the research to critical review, we may take the short cut by jumping to conclusions not necessarily merited by the research. This problem can be compounded when the research is flawed, and the invalid conclusions, rather than adding to the discussion about best practices in mathematics, create confusion and misperceptions.

What we all need—teachers and students alike—is sound research on math, from which we can establish the most effective interventions for our students. What is *not* needed is flawed math research which is incorrectly structured, or disassociated from research on neuroscience or cognitive science, particularly if the invalid conclusions from this research discredit already-proven methodologies.

An example of the above is Peg W. Bogle's study, "From Learning Multiplication Facts to Automaticity: A Comparison of Two Methods in Students with Learning Disabilities," in the Winter 2008 issue of the AET journal, *The Educational Therapist,* Volume 29, number 1, on pages 5-9. In her study, Bogle attempts to compare the 9 Lines strategy with the use of factor times factor flash cards. Bogle's study lacks structure and organization. In her study, Bogle misunderstood and misapplied the 9 Lines strategy to such an extent that her resulting version of it does not represent the actual strategy. In conjunction with her misapplication of the 9 Lines strategy, the flaws in her design and protocol significantly affect the validity of her conclusions. Bogle's study lacks the fundamentals of basic research, as her experiment does not exhibit the planning and structure of a researcher who understands the essential elements of a study. Her experiment would have been greatly improved if she had done the following:

1. Established equivalence in the study. The 9 Lines strategy and flash cards are not equivalent methods. In addition, she should have created equivalent groups of students that would all have been in the same grade and grouped by assessment to determine equivalence of previous instruction, processing styles, and baseline skills levels.
2. Designed an experiment with controlled variables to support internal validity.
3. Applied correct statistical measures with much smaller standard deviations and greatly reduced overlap between group scores.
4. Used a significantly larger sample size than 15. A recommended minimum would be 100 students.
5. Expanded the study to include full automaticity of retention and application with the 100 multiplication facts, not just memorizing the products of a single times table.
6. Designed a longitudinal study of at least three years to accurately measure full development of automaticity.
7. Completed the 9 Lines intensive course, which provides the correct structure and application of the strategy.
8. Acquired expertise with the 9 Lines multiplication fact acquisition and application strategy, including significant supervised practice while working with diverse students' processing styles, prior to including the strategy in research.
9. Correctly applied the 9 Lines strategy by including all of the components, maintaining the multisensory structure, and using the specific receptive and expressive language throughout.
10. Based her experiment on direct connections to the neuroscience and cognitive science of automaticity.
11. Been more cautious about asserting conclusions prematurely.

As the creator and founder/director of Making Math Real, I have been developing the multisensory structured 9 Lines strategy for more than 32 years. This strategy is a multisensory structured intervention requiring significant training and subsequent supervised practice in order to apply correctly and effectively. As with all multisensory structured practices, teaching the 9 Lines strategy requires the practitioner to be highly experienced in the methodology and to utilize the precise structure in its design and implementation. If any structural elements are omitted, altered, and/or incorrectly applied, the outcome of the strategy is subverted.

Bogle's understanding of the 9 Lines strategy is nominal, limited only to a brief demonstration she experienced in a two-day seminar entitled Making Math Real: Overview in 2005. The Overview seminar is a broad introduction to the pedagogical foundations and research that support the 320 hours of the subsequent course content that compose the Making Math Real methodologies, K-12. Practitioners in the course are advised that the Overview demonstration is only an introductory experience. Therefore, Bogle had not received requisite training or experience in the 9 Lines strategy prior to her research.

Bogle admits in the discussion section of her study: "In order to make the interventions as equivalent as possible in terms of the number of exposures to each fact per session, the length of time of each session, the intervention procedures and the post-test format, the 9 Lines™ intervention method was streamlined in ways that may have rendered it a less effective tool." Bogle's admission of streamlining the 9 Lines strategy is vastly understated. To appreciate the extent to which Bogle misapplied my method, it is crucial to critically pinpoint the incorrect protocol of her study by contrasting it with the actual strategy as it is taught in the 9 Lines intensive course.

The 9 Lines strategy is an intensive and highly structured multisensory intervention, the design and purpose of which is to flood the visual, auditory, and kinesthetic-motoric modalities, focusing on the perceptual and associative sensory-cognitive tools of central processing, in order to maximize the imaging and storage of symbolic information. The strategy is expressly designed to develop students' symbol imaging for mathematical symbols, not to teach the concepts of multiplication. Therefore, the four concepts of multiplication—grids and arrays, repeated addition, grouping, and skip counting—must be taught to students prior to using the 9 Lines strategy. Symbol imaging is a sensory-cognitive tool for mentally perceiving, holding, storing, and retrieving sequences of numbers. My experience, research, and assessment indicate a high correlation between symbol imaging development and automaticity with *all* of the math facts, not just the multiplication facts. The development of symbol imaging and automaticity for the student with learning disabilities frequently requires three years of prescriptive intervention. The 9 Lines strategy is designed only for imaging the multiplication and division facts. There are distinct interventions for imaging the addition and the subtraction facts.

Students with underdeveloped symbol imaging face significant challenges in accessing memory for math facts. Their ability to process information encoded in symbols ($6 \times 7 = 42$, $8 + 6 = 14$) is directly affected. If students cannot mentally image all of the symbols in a number sequence, they will not be able to store it in its intact form in memory. If the memory itself is not completely intact, then it is not possible to retrieve it accurately later from long-term memory. Students who cannot perceive all the symbols in math facts frequently rely on compensatory strategies that require counting. Rather than mentally reconstructing a math fact, students lacking symbol imaging typically have to count each time, making physical attempts to keep track of their counting as they touch parts of their bodies, make tally marks, or make dots. Developed symbol imaging allows the individual to mentally reconstruct the math fact without having to count. The reliance on counting to solve math facts often results in diminished calculation accuracy and reduced cognitive endurance, while mental reconstruction of math facts reflects higher cognitive efficiency and reduces the cognitive load.

Symbol imaging is developed by guiding students to image a multisensory perceptual picture of a group of symbols, such as a times table. The educator then systematically deconstructs the symbol-based picture, one symbol at a time, while structuring the student to systematically reconstruct each and all of the missing symbols until the student can successfully reconstruct the factors and their products in any order. The specific design of the 9 Lines multiplication fact acquisition and application strategy guides students systematically through the experience of imaging, deconstructing, and reconstructing. The entire process is composed of four distinct phases, each including assessment points with which to monitor students' success at each phase. It is imperative not to omit any of the four phases or their respective assessment points.

Phase one consists of guiding students to use visual and auditory sequential memory in order to take an initial perceptual image of the times table. To perform phase one correctly, the student needs to be seated in front of a large whiteboard on which the times table will be recorded above the student's eye level, an approach that supports the neuro-linguistic programming of moving the eyes upwards to access visual memory. The educator and student construct each multiplication fact together by saying each fact aloud in a choral fashion while the educator records each fact at a time within a prescribed three-column, three-row format. When the entire table is recorded, formal imaging begins. The student is directed to say only the products, not the factors, in order from beginning to end three times over. They are directed to say only the products because, lacking sufficient symbol imaging development, they are unable to perceive all of the symbols in an entire math fact. However, students are able to image the one or two digits comprising all of the multiplication facts' products. The structure of the four phases of the 9 Lines strategy will reconnect the products to their respective factors in phase four, the applied practice section. During this time the educator points and taps, indicating each product as the student recites. Guiding the student's visual processing in this manner helps maximize the perception of each product, and moreover, the student's experience of "following the bouncing ball" as the educator points and taps, stimulating the motor cortex. After three successful recitations, the educator systematically erases one product (not the factors—the factors remain with a blank space after them to indicate the missing product) in a prescribed order. To facilitate successful imaging, the educator helps the student "take one last picture before each product is removed." As each product is erased, the student is directed to say all the products in order, including the one(s) that are no longer there. Through the repeated practice of saying the products in order, the student uses visual and auditory sequential memory to reconstruct each missing product from the table. However, and most importantly, after each successful reconstruction of the table, the educator directs the student back to each missing product and prompts the student to immediately reconstruct only the missing products in a random order. To further support students' ability to reconstruct products in random orders, the educator applies additional auditory mnemonics to help focus perception on the location or relationship of the products. For example, in the three-column, the three-row structure of the 9 Lines strategy, the ✕ 5 product is located in the middle, and is referred to as "the middle-middle product." The student's ability to successfully reconstruct each product disassociated from its sequential order takes the student beyond a reliance on sequential order (skip counting) to re-image a product, to the immediate symbol imaging recall of any given multiplication fact. In her research, Bogle omitted this symbol imaging development in her misapplication of the 9 Lines Strategy during this phase, and did not provide the auditory mnemonics to support the location of a given product.

Once all nine of the products have been erased, the student has successfully reconstructed the table in sequential order nine times. Therefore, the students are now engaged with a times table wherein all the products are missing, prompting the first of four assessment points. Each of the assessment points provides essential indications whether the student is ready to continue or require further imaging intervention. The first assessment point measures the student's ability to successfully use visual and auditory sequential memory to reconstruct the products in order. Reconstructing the products in order is the initial development in support of the students' ability to reconstruct the products in a random order. If students can successfully reconstruct all missing products in order, then assessment point number two is indicated.

Assessment point number two has students reconstruct each missing product again, but this time in a random order. Assessment point number two measures the student's ability to begin moving beyond skip counting and using symbol imaging directly to immediately recalling missing products. The educator directs the student to reconstruct each missing product in a random order by giving four processing cues to support success:

1. Point to the indicated missing product.

2. Tap while pointing.
3. Give location: “middle-middle,” “biggest in the house,” etc.
4. Say the factors: “seven times five,” “seven times nine,” etc.

A complete prompt would have the educator pointing, tapping, and saying, “middle-middle, seven times five, how much is that?” As the student correctly names each product, the educator writes the product back on the board until all missing products have been reconstructed in written form.

When assessment point number two is complete, the second phase of the 9 Lines strategy begins. In her research, Bogle did not employ any component of phases two through four of the 9 Lines strategy, limiting her research to the misapplication of phase one. In the discussion section of her study, Bogle refers to the omission of phases two and three as an “additional 8 practice exposures”, and incorrectly states their method for imaging and integration. She also refers to the omission of phase four—the phase for developing automaticity—without understanding its vital role. Bogle writes: “In *Making Math Real*, Berg (2005) extends the teaching of the math facts using the 9 Lines™ with an additional 8 practice exposures over and above the 13 exposures in the 9 Lines™ intervention used in this study. In Berg’s curriculum for 9 Lines™ students use the same procedure for the first 13 exposures used here but then go on to practice the multiplication facts in vertical and horizontal formats and with varied missing factors. ... Berg states that ‘after successfully synthesizing the multiplication facts from a particular times table, students have created intact visual files.’ This does not mean that they have necessarily established automaticity. The process of building automaticity continues with the consistent application of mentally re-imaging the facts and repeatedly storing and retrieving them from memory.”

Bogle’s omission of phases two through four is the most egregious transgression of her misapplication of the 9 Lines strategy for the following reasons:

- Omitting phase two eliminates the motor component.
- Omitting phase three eliminates the association of the imaged products to the 9 Lines mnemonic.
- Omitting phase four eliminates the prescriptive applied practice to develop automaticity.

To facilitate maximum integration, phase two of the 9 Lines strategy integrates students’ gross motor skills along with visual and auditory sequential memory to re-image the products in order. The educator provides all the factors without their respective products on the board in the same three-column, three-row format as before, but in a sufficiently large scale that incorporates gross motor modality in the physical reconstruction of each product. The student stands at the board with a dry erase marker and eraser, and is instructed to say and write all the missing products in order. After each successful reconstruction, the educator acknowledges the student’s good work, and has the student erase only the products, and then repeat the process. When the student has completed three successful reconstructions, assessment point number three begins.

This assessment point measures the student's ability to image and reconstruct all of the products in a random order, successful reconstruction in a random order indicates that the student has established sufficient symbol imaging for each product in the table and will be ready for the third phase of the 9 lines. When administering the assessment, the educator prompts the student to say and write the products in a random order in the following way: "Hardest in the house, seven times seven, how much is that? The baby, seven times one, how much is that? Biggest in the house, seven times nine, how much is that…"

Phase three guides students to associate their successful symbol image of the products to a mnemonic device, the 9 lines. For phase three, the 9 lines are presented on the whiteboard in the same three-column, three-row format as were the factors and their products, and students are taught that each of the 9 lines represents each of the factors and their products. The first line represents $7 \times 1 = 7$, the second line represents $7 \times 2 = 14$, all the way through to $7 \times 9 = 63$. The student is prompted to say and write only the products in order. After each successful reconstruction the educator acknowledges the correct responses and has the student erase the products, and say and write the products in order again. When the student has completed three successful reconstructions, the final assessment—assessment point number four—begins.

The final assessment determines successful association of the products with their respective locations on the 9 Lines mnemonic device. As before, the educator's prompting provides the student with location clues and the specific factors in a random order. The educator has the student say and write each product where it is located on the 9 lines. After the successful association of the products to the 9 lines, students are ready for phase four. It should take the experienced educator approximately 20 minutes to guide students from phase one through phase three. Phase four, the applied practice, is the most essential component for developing automaticity. The applied practice phase has students consolidate the multiplications facts in at least three formats:

1. Factor times factor.
2. Missing factor.
3. Reconstruct the factors from given products.

All practice formats are presented in random order in vertical format, in number sentence format, and with the equalities reversed. All practice worksheets include a blank 9 lines for the student to use as a mnemonic support. The student is directed to leave the 9 lines blank and not to write the products back on the mnemonic. The educator explains that by leaving the 9 lines blank, the student will build symbol imaging by reconstructing products mentally, because copying products from a filled-out 9 lines inhibits symbol imaging development. In her research, Bogle further subverted the integrity of the 9 lines strategy by omitting phase three and eliminating the participants' association of the products with their prescriptive mnemonic, the blank nine lines. In the discussion section of her study Bogle says, "Berg also uses a mnemonic of a blank 9 lines™ format on facts worksheets and practice tests to aid students in recalling the multiplication products. The mnemonic is nine blank lines, three across horizontally, in three rows that represents the nine math facts in the 9 lines™ format. The 9 lines™ mnemonic was not present on the post-test sheet of math facts as a visual aid for the participants in the 9 lines™ intervention group."

The applied practice develops automaticity by having the students re-image, store, and retrieve the symbol image of the multiplication facts in any and all configurations. In addition to the prescriptive worksheet practice of phase four, and because the student must re-image, store, and retrieve the facts daily, even if only for five to ten minutes, the "Eight Levels of Daily Re-imaging" is the most commonly used specific prescriptive practice for developing automaticity with the multiplication facts. If the student does not re-image the facts on a daily basis, significant visuo-perceptual loss may occur, and the development of symbol imaging and automaticity will be unnecessarily protracted.

The "Eight Levels of Daily Re-imaging" is designed for the student to perform only as an oral response, and not as a written one. The educator focuses the student's attention on a blank 9 lines mnemonic representing a specific times table. The 9 lines remain blank during the entire 5- to 7-minute activity. In levels one through seven, the educator systematically guides students to re-image, store, and retrieve the facts in all factor-product configurations including connections to the division facts. Level eight extends the basic multiplication facts by having the educator prompt students through all levels, 1–7, but not to reconstruct the 7s, for example, but rather to extend the 7s to the 70s and/or the 700s.

SYNTHESIS

In review, the 9 Lines Strategy is an intensive, systematic, multisensory structured intervention composed of four phases and four assessments. It takes the well-practiced educator twenty minutes to complete phases one through three. The first two phases help the student symbol image the products, the third phase associates the products with a perceptual organizing mnemonic, and the fourth phase develops automaticity through prescriptive applied practice. The ongoing integrated assessment points provide the educator with critical information measuring success of imaging and subsequent association of the imaging with the mnemonic.

In contrast, Bogle's study offers partial and incorrect components of phase one as the sole representation of the 9 lines strategy. In the methodology section of her study, Bogle states: "Each participant then met with the researcher for up to seven interventions to practice the 7s math facts using either flashcards ... or the 9 lines™ method. In every intervention session the participant was exposed to each of the facts 13 times. The intervention sessions for both methods lasted approximately 8-12 minutes but were not timed. Sessions for all participants were conducted over an 8-week period during the regular school day, usually during math class. There was no attempt to control the length of time between sessions or between the last intervention session and the subsequent posttest, and later, the delayed posttest. The delayed posttest was administered between 14 and 36 days after the posttest."

This limited version does not include the culminating experience of phase one, the fourteenth exposure. The fourteenth exposure refers to assessment point number two, where the student reconstructs all the products in a random order, thereby measuring initial symbol imaging of the products. Furthermore, during exposures five through thirteen, Bogle did not have participants go back after each successful reconstruction in sequential order to immediately reconstruct each missing product in random order, a required component of developing symbol imaging in phase one. In addition, Bogle refers to "exposures" as the principal method of intervention for the 9 lines strategy. An exposure is a passive, receptive experience only, and does not engage the perceptual and associative sensory-cognitive tools of central processing that support imaging, storage, and retrieval. The 9 lines strategy is designed to actively engage receptive and expressive visual, auditory, and kinesthetic-motoric processing to maximize imaging integration and consolidation. As a multisensory structured intervention, the inappropriate instructional technique of "exposing" students does not occur at any time during the 9 lines strategy or in any component of the Making Math Real methodologies, K-12.

Finally, Bogle's research exhibits significant design flaws. The high incidence of uncontrolled variables and the lack of equivalence throughout the study subvert the internal validity of her research. In the methodology section of her study, Bogle states, "The methodology for this study was implemented with the acknowledgement of several limitations. First, by its nature, it was a small sample, limited to a total population of less than 20 (actual number was 15 participants) students in third and fourth grade, at one urban independent school for students with learning disabilities. Actual students available to participate were further limited to those who were given parental permission to participate. Further, the intervention sessions had to be limited to no more than about twelve minutes, in order to minimize the amount of time students were pulled out from regular math or other instruction, and to ensure that data collection could be completed in a timely manner. Participant availability on any given data collection day was also affected by school schedule, participant absences, field trips, teacher requests, and researcher availability. In addition, there was no effort to control what participants would learn and practice or not learn and practice outside the intervention sessions which might impact their performance on post-tests. Nor was there a control group in this study."

CONCLUSION

In summation, Bogle observed a demonstration of the 9 lines strategy intervention in an overview course and, based on this cursory exposure, decontextualized it and misapplied it in a research project. Yet Bogle asserts in the conclusion section of her study, "Overall, this study provides significant results to educational therapists who take on the goal of teaching math facts to their students. The group of students taught with flashcards outperformed the group of students taught with 9 lines™."

Since the students in Bogle's study were never taught the actual 9 lines multiplication fact acquisition and application strategy, her conclusions about its relative effectiveness are misleading. Bogle acknowledges limitations in the study, yet the limitations she notes do not accurately reflect the full extent to which she misapplied the 9 lines strategy, misused statistical data, and failed to establish equivalence between the strategies and groups compared.

Beyond the limitations of Bogle's study identified in this rebuttal, there are more significant limitations to consider. For example, her study looks only at learning a single times table. The cognitive demands of learning all the times tables and developing automaticity across the tables for the 100 multiplication facts requires a much more intensive and comprehensive intervention than is needed for learning a single table. In addition, it is not sufficient to learn the multiplication facts only in the flash card prompt of factor times factor configurations. The most valuable mathematical development in learning the times tables is the ability to apply the multiplication facts in diverse ways, such as solving for missing factors and finding the greatest common factor of two products. Students must be able to identify any and all parts of all fact families in any order to support multiplication and division with whole numbers, fraction equivalence and lowest terms, the four operations with fractions and decimals, and factoring whole numbers and polynomials. I have designed the 9 Lines Multiplication Fact Acquisition and Application Strategy specifically to develop the symbol imaging and the mental organizational structures that support the full application of the multiplication facts.

Flash card practice is useful within a comprehensive, long-term intervention for developing automaticity if it is introduced at the correct time and in the correct manner. Flash card use is indicated when students, through the prescriptive interventions of the 9 Lines strategy structure, have generated sufficient symbol imaging to perceive all of the symbols on the flash card. I use flash cards as part of the applied practice phase of the 9 Lines strategy, specifically in levels 4 through 7 of the 8 levels of daily re-imaging. However, flash cards only present a given fact without any structure to support the application of the fact and are therefore insufficient as a stand-alone method for developing automaticity. I have frequently observed students "memorize" a fact from a flash card but be subsequently unable to apply that fact when it was embedded within a problem or if it was presented in a different configuration. Furthermore, exposure to flash cards provides no system for students to recall a forgotten fact. The only way to help the student is by showing the student the flash card again. This repeated exposure does nothing to develop the processing tools that support consolidating the forgotten fact in long-term memory. Consequently, students are shown the cards again and again, and they still cannot retain or apply the fact. The 9 lines method is designed to develop the processing tools that support imaging, storage, and retrieval, and it provides students with a mnemonic to aid recall.

Since 1996, I have been providing Making Math Real professional development to public and private schools nationwide. The most pervasive problem expressed by teachers, from first grade through college, has been the need to effectively teach the math facts. According to my research and practice, not only students with learning disabilities, but at least 50 percent of all students nationwide have not mastered the math facts. Flash cards have been used for generations, and clearly are not a sufficiently robust methodology to develop automaticity—a fact that any experienced classroom teacher understands only too well.

I agree with Bogle when she states in the discussion section, “Further research into the efficacy of other visual instruction strategies such as the 9 Lines for achieving math fact automaticity would be of considerable value in validating this method.” However, the neuroscience and cognitive science of developing automaticity for the math facts remains relatively unexplored, even by the most established and experienced researchers. Therefore, I would strongly recommend any future study involving the 9 Lines Strategy to exercise caution by waiting until sound research on the neuroscience and cognitive science supporting automaticity is established. The 9 Lines Strategy needs to be applied by a trained expert.

Since symbol imaging for students with learning disabilities frequently takes up to three years to develop, the study needs to be longitudinal, covering a minimum of three years, to accurately measure full automaticity with all the facts, not merely a single times table, again requiring the skills of an experienced researcher.

Math educators are underserved by research, especially when the research presents conflicting results. Thousands of teachers and students have experienced the unprecedented success of the 9 Lines Strategy, and as the developer of the method it is my intention that its success continues with the support of scientific validation from experienced educational researchers.

For further information on Making Math Real, please see www.makingmathreal.org.

David Berg is an educator of more than 33 years, having taught in grades K through postgraduate in general education, bilingual education, special education, and educational therapy. He was an instructor in the Educational Therapy Certificate Program at UC Santa Cruz Extension and a member of the program’s advisory board. In addition, Mr. Berg was a cofounder of the certificate program at UC Berkeley Extension, was one of the key instructors in that program, and was a member of the program’s advisory board. Mr. Berg is the creator of the Making Math Real Multisensory Structured Methodologies, K-12, and is the founder/director of the Making Math Real Institute. He is also an author, lecturer, and professional development trainer at universities, conferences, and schools throughout the nation.